Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Engineering Mathematics-II

Subject Code: 4TE02EMT1 Branch: B.Tech(All)
Semester: 2 Date: 09/05/2016 Time: 10:30 to 1:30 Marks: 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) A square matrix A is called symmetric matrix if

(a) $A^T = -A$ (b) $A^2 = A$ (c) $A^T = A$ (d) $A^2 = I$

b) The determinant of the matrix $\begin{bmatrix} 1 & 5 & 3 \\ 0 & -2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 6 (d) - 6

c) A $n \times n$ Non-Homogeneous system of equations $AX = B$ is given. If $\rho(A) = \rho(A:B) < n$ then the system has

(a) No solutions (b) Unique solutions
(c) Infinite solution (d) None of these

d) The sum of the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 6 (d) – 6

e) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 3 (d) 0

f) $\int_0^2 \int_2^4 \int_4^6 dx\,dy\,dz =$ _____

(a) 1 (b) 6 (c) 4 (d) 8

g) $\int_0^1 \int_0^{\sqrt{x}} dy\ dx =$ _____

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 0 (d) y

h) $\int_{-\pi/2}^{\pi/2} \sin^9 x\ dx =$ _____

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

i) The value of $\int_{-\pi}^{\pi} \sin mx \sin nx\ dx$ for $m \neq \pm n$ is

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0

j) $\int_1^{\infty} \frac{1}{x^2}\ dx$ is

(a) Converges (b) Diverges (c) Oscillatory (d) None of these

k) The order of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}}$ is

(a) 1 (b) 2 (c) 3 (d) 6

l) The equation $P(x,y)\,dx - Q(x,y)\,dy = 0$ is exact if

(a) $P_x = Q_y$ (b) $P_y = Q_x$ (c) $P_x = -Q_y$ (d) $P_y = -Q_x$

m) A vector $\vec{F}$ is said to be solenoidal if

(a) $\nabla \times \vec{F} = 0$ (b) $\nabla \cdot \vec{F} = 0$ (c) $\nabla \times \left(\nabla \cdot \vec{F}\right) = 0$ (d) None of theser

n) If $r = xi + yj + zk$ then div r is

(a) 0 (b) r (c) 3 (d) – r

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. **(05)**

b) Evaluate: $\int_0^{\pi} x \sin^8 x \cos^6 x\ dx$ **(05)**

c) Solve: $\frac{dy}{dx} - \frac{3y}{x} = x^3,\ y(1) = 4$ **(04)**

Q-3 Attempt all questions

a) Find the inverse of the matrix $A=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix}$ by using determinant method. **(05)**

b) Solve the following system of equations by Cramer's rule: **(05)**

$$x+y+z=6;\quad x+2y+3z=14;\quad x+4y+9z=36$$

c) Reduce the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 5\\ 4 & 5 & 6 & 7\\ 11 & 12 & 13 & 14\end{bmatrix}$ to the normal form and find its rank. **(04)**

Q-4 Attempt all questions

a) Find the inverse of the following matrix by using elementary transformation **(05)**

$$A=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$$

b) Obtain Reduced row echelon form of the following matrix: **(05)**

$$A=\begin{bmatrix}1 & 3 & 2 & 1\\ 2 & 3 & 3 & 2\\ 3 & 4 & -1 & 3\\ 6 & 10 & 4 & 6\end{bmatrix}$$

c) Solve the system of equation **(04)**

$$x+2y-z=3;\quad 3x-y+2z=1;\quad 2x-2y+3z=2;\quad x-y+z=-1$$

Q-5 Attempt all questions

a) Evaluate: $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$ **(05)**

b) Solve: $\frac{dy}{dx}+x\sin 2y=x^2\cos^2 y$ **(05)**

c) Evaluate $\int_C \bar{F}\,d\bar{r}$ along the parabola $y^2=x$ between the points $(0,0)$ and where $\bar{F}=x^2\,\hat{i}+xy\,\hat{j}$. **(04)**

Q-6 Attempt all questions

a) Find the eigenvalues & eigenvectors of a matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$. **(05)**

b) Show that $\bar{F}=\left(y^2-z^2+3yz-2x\right)\hat{i}+\left(3xz+2xy\right)\hat{j}+\left(3xy-2xz+2z\right)\hat{k}$ is both solenoidal and irrotational. **(05)**

c) Define: Gradient and find $\nabla\phi$ at $(1,-2,1)$, if $\phi=3x^2y-y^3z^2$. **(04)**

Q-7 Attempt all questions

a) Change the order of integration and evaluate $\int_0^a\int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}}\left(x^2+y^2\right)dx\,dy$. **(05)**

b) Find the area bounded by the parabola $y^2=4x$ and the line $2x-3y+4=0$. **(05)**

c) Solve: $2xy\,dy+\left(x^2+y^2+1\right)dx=0$ **(04)**

Q-8 Attempt all questions

a) State and verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$. **(07)**

b) Verify Green's theorem for $\oint_C\left[\left(x^2-2xy\right)dx+\left(x^2y+3\right)dy\right]$ where C is the boundary of the region bounded by the parabola $x^2=y$ and the line $x=y$. **(07)**